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A Study on Double General Integral Transform For Solving Hyperbolic Boundary Value Problem

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ABSTRACT
In this paper, we use formulas and properties of parabolic boundary value problem and implement them to solve hyperbolic boundary value problems using double general integral transform. These problems typically have initial conditions along with boundary conditions specified at different points in the domain. Solving hyperbolic boundary value problems can be challenging due to the complex nature of the equations involved.

Key Words and Phrases: Double General Integral Transform, Ordinary Differential Equation, Partial Differential Equation, Parabolic Boundary Value Problem, Hyperbolic Boundary Value Problem, Inverse Transform.

1. Introduction

Transforms have been successively used since the 1780s. [7] Many integral transforms such as Laplace transform, Sumudu transform, Aboodh transform, Elzaki transform, Maghoub transform, Natural transform, and Kamal transform have been developed by many academicians to solve Ordinary differential equations, Partial differential equations, Integral equations, Integro-differential equations, system of differential equations, boundary value problems[1].

An Integral transform is a Mathematical technique that converts a given function or data from one domain into another by using integral equations[14]. Integral transforms, help to change the representation of functions, making it easier to analyze complex systems and solve various types of Differential Equations. They often shift the problem from one domain to a different domain, where the analysis or solution may be more convenient[11]. Integral transformations is a really effective way to solve Partial differential equations[2]. Partial differential equation are math tools that help describe a lot of things in physics and other sciences, making them super useful. They're like magic tools that turn complex equations into simpler ones for easier solving[4]. General integral transform is given as:

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$$T\{f(t)\} = m(s)\int_0^\infty f(t)e^{-n(s)}dt$$

2. Preliminaries

2.1. Double General Integral Transform

A double general integral transform is a mathematical operation that converts a two-dimensional function into a different representation, often used for solving complex problems in mathematics and engineering. It is defined as:
Let f (x, y) be an integrable function defined for the variables x and y in the first quadrant $p_1(s) \neq 0$, $p_2(s) \neq 0$ and $q_1(s)$, $q_2(s)$ are positive real functions; we define the double general integral transform $T_2\{f(x, y)\}$ by the formula
Provided that the integral exists for some $q_1(s)$, $q_2(s)$.
[1]The double general integral transform is a valuable tool for solving boundary value problems in Partial Differential Equations, including elliptic, parabolic, and hyperbolic problems[4]. Researchers are actively exploring different integral transforms to address these issues, and the double general integral transform has been used to solve parabolic boundary value problems, particularly those involving heat equations. This approach [6] has proven to be effective and useful in obtaining solutions for parabolic boundary value problem.

2.2 Properties of Double General Integral Transform

2.2.1 Linearity Property

$$T2\{af(x,y) + bg(x,y)\} = aT2\{f(x,y)\} + bT2\{g(x,y)\}$$

Proof

L.H.S $= T2\{af(x,y) + bg(x,y)\}$

$$= m_1(s)m_2(s)\left[\int_0^\infty\int_0^\infty e^{-(n_1(s)x+n_2(s)y)}\, af(x,y)dxdy + \int_0^\infty\int_0^\infty e^{-(n_1(s)x+n_2(s)y)}\, bg(x,y)dxdy\right]$$

$$= am_1(s)m_2(s)\int_0^\infty\int_0^\infty e^{-(n_1(s)x+n_2(s)y)}\, af(x,y)dxdy + {}^{b}am_1(s)m_2(s)\int_0^\infty\int_0^\infty e^{-(n_1(s)x+n_2(s)y)}\, bg(x,y)dxdy$$

$=aT2f(x,y) + T2g(x,y)\}$

$=$R.H.S

2.2.2 Shifting Property.
If $T_2\{f(x,y)\} = \tau(s)$ then
$T_2\{e^{ax+by} f(x,y)\} = \tau(s,a,b)$
$= m_1(s)m_2(s) \int_0^\infty \int_0^\infty e^{-(n_1(s)+a)x+(n_2(s)+b)y} f(x,y)dxdy$

Proof
$T_2\{e^{(ax+by)} f(x,y)\} = \tau(s,a,b)$
That is
$T_2\{e^{-(ax+by)} f(x,y)\} =$
$m_1(s)m_2(s) \int_0^\infty \int_0^\infty e^{-(n_1(s)+a)x+(n_2(s)+b)y} f(x,y)dxdy$

L.H.S $= T_2\{e^{-(ax+by)} f(x,y)\}$
$= m1(s)m2(s) \int_0^\infty \int_0^\infty e^{-(n1(s)x + n2(s)y)} + e^{(ax+by)} f(x,y)dxdy$
$= m1(s)m2(s) \int_0^\infty \int_0^\infty e^{-(n1(s)x + n2(s)y + ax + by)} f(x,y)dxdy$
$= m1(s)m2(s) \int_0^\infty \int_0^\infty e^{-(n1(s)+a)x + (n2(s)+b)y} f(x,y)dxdy$
R.H.S

2.2.3 Change of Scale Property.
If $T_2\{f(x,y)\} = \tau(s)$ then $T_2\{f(ax,by)\} = \frac{1}{ab}\tau(s,a,b)$

Proof
L. H.S $= T_2\{f(ax,by)\}$
$T_2\{f(x,y)\} = m_1(s)m_2(s) \int_0^\infty \int_0^\infty e^{-(n1(s)x + n2(s)y)} f(ax,by)dxdy$
substituting $ax = u$ and $by = v$ we get $x \to 0,\ u \to 0$ and $y \to 0,\ v \to 0$ also as $x \to \infty,\ u \to \infty$ and $y \to \infty,\ v \to \infty$
$adx = du \Rightarrow dx = \frac{du}{a}$ and $bdy = dv \Rightarrow dy = \frac{dv}{b}$
Put value in above equation

$$= m_1(s)m_2(s) \int_0^\infty \int_0^\infty e^{-\left(\left(\frac{n1(s)}{a}\right)u + \left(\frac{n2(s)}{b}v\right)\right)} f(u,v)\, du\, dv$$

$$= \frac{1}{ab}\left[m1(s)m2(s) \int_0^\infty \int_0^\infty e^{-(r_1(s))u + (r_2(s)v)} f(u,v)\, du\, dv\right]$$

$= \frac{1}{ab}\tau(s,a,b)$
$=$ R.H.S

3. Proof of the Main Results

We will demonstrate the primary finding from part one in this section. But for this, we need a few helpful theorems[2].

3.1. Theorem. Let $f(x,y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and $f(0,y)$ be given $m_1(s)$,and $n_1(s)$ are

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Vol. 3 No. 12 (December) (2025)

positive

real functions then

$T_2\{\frac{\partial f}{\partial x}(x, y)\} = -m_1(s)T\{f(0, y)\} + n_1(s)T_2\{f(x, y)\}$
where $T\{f(0, y)\}$ is the new general integral transform of the $f(0, y)$.

3.2 Theorem. Let $f(x, y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and $f(x, 0)$ be given $m_2(s)$ and $n_2(s)$ are positive real functions then

$T_2\{\frac{\partial f}{\partial x}(x, y)\} = -m_2(s)T\{f(x, 0)\} + n_2(s)T_2\{f(x, y)\}$
where $T\{f(x, 0)\}$ is the new general integral transform of the $f(x, 0)$.

Function $f(x, y)$	Double General Integral Transform $T_2\{f(x, y)\}$
1	$\frac{m_1(s)m_2(s)}{n_1(s)n_2(s)}$
$e^{(ax+by)}$	$\frac{m_1(s)m_2(s)}{(n_1(s)-a)(n_2(s)-b)}$
$e^{(\iota(ax+by))}$	$\frac{m_1(s)m_2(s)}{(n_1(s)-\iota a)(n_2(s)-\iota b)}$
$cosh(ax+by)$	$\frac{1}{2}[\frac{m_1(s)m_2(s)}{(n_1(s)-a)(n_2(s)-b)} + \frac{m_1(s)m_2(s)}{(n_1(s)+a)(n_2(s)+b)}]$
$sinh(ax+by)$	$\frac{1}{2}[\frac{m_1(s)m_2(s)}{(n_1(s)-a)(n_2(s)-b)} - \frac{m_1(s)m_2(s)}{(n_1(s)+a)(n_2(s)+b)}]$
$cos(ax+by)$	$\frac{1}{2}[\frac{m_1(s)m_2(s)}{(n_1(s)-\iota a)(n_2(s)-\iota b)} + \frac{m_1(s)m_2(s)}{(n_1(s)+\iota a)(n_2(s)+\iota b)}]$
$sin(ax+by)$	$\frac{1}{2}[\frac{m_1(s)m_2(s)}{(n_1(s)-\iota a)(n_2(s)-\iota b)} - \frac{m_1(s)m_2(s)}{(n_1(s)+\iota a)(n_2(s)+\iota b)}]$
$(xy)^q, q > 0$	$\frac{(\Gamma(q+1))^2 m_1(s)m_2(s)}{(n_1(s)n_2(s))^{k+1}}$
$x^p y^q, p > 0, q > 0$	$\frac{\Gamma(p+1)\Gamma(q+1)(m_1(s)m_2(s))}{(n_1(s))^{p+1}(n_2(s))^{q+1}}$

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Vol. 3 No. 12 (December) (2025)

3.3 Theorem. Let (x, y) be a function of two variables. If the first and second ordered partial derivative $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$and $\frac{\partial^2 f}{\partial y^2}$ exist and $f(0, y)$, $f_x(0, y)$be given $m_1(s), m_2(s), n_1(s)$ and $n_2(s)$ are positive real functions then

$T_2\{ \frac{\partial^2}{\partial x^2} f(x, y)\} = -m_1(s)[T\{f_x(0, y)\} + n_1(s)T\{f(0, y)\}]$
$+ n1^2(s)T_2\{f(x,y)\}$

where, $T\{f_x(0, y)\}$, $T\{f(0, y)\}$ is the new general integral transform of the $\{f_x(0, y)\}$, $f(0, y)$ respectively.

3.4 Theorem. Let (x, y) be a function of two variables. If the first and second ordered partial derivative$\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$and $\frac{\partial^2 f}{\partial y^2}$ exist and $f(x, 0)$, $f_x(0, y)$be given $m_1(s), m_2(s), n_1(s)$ and $n_2(s)$ are positive real functions then

$T_2\{ \frac{\partial^2}{\partial y^2} f(x, y)\} = -m_2(s)[T\{f_y(x, 0)\} + n_2(s)T\{f(x, 0)\}] + n_1(s)\ T_2\{f(x, y)\}$

where, $T\{f_y(x, 0)\}$, $T\{f(x, 0)\}$ is the new general integral transform of the $\{f_y(x, 0)\}$, $f(x, 0)$ respectively.

4. APPLICATION

The DGIT will be used in this section to address the Hyperbolic Boundary Value Problem[3].

4.1 Example. $\{\frac{\partial^2 u}{\partial x^2}\} + \{\frac{\partial^2 f}{\partial y^2}\} = 0$ under some restrictions. $u(x, b) = 0$ $u(a, y) = 0$ $u(0, y)$ is equal to 0. and $u(x, 0) = \sin x$

Solution

$T\{\frac{\partial^2 u}{\partial x^2}\} + T\{\frac{\partial^2 f}{\partial y^2}\} = 0$

$L.H.S = T_2\{ \frac{\partial}{\partial x} u_x(x, y)\} + T_2\{ \frac{\partial}{\partial y} u_y(x, y)\}$

$= -m_1(s)T\{u_x(0, y)\} + n_1(s)T_2\{u_x(x, y)\}$
$-m_2(s)T\{u_y(x, 0)\} + n_2(s)T_2\{u_y(x, y)\}$
$= n_1(s)T_2\{u_x(x, y)\} + n_2(s)T_2\{u_y(x, y)\}$
$+n_1(s)T_2\{ \frac{\partial u}{\partial x}(x, y)\} + n_2(s)T_2\{ \frac{\partial}{\partial y} u(x, y)\}$

$= n_1(s)[-m_1(s)T\{u(0, y)\} + n_1(s)T_2\{u(x, y)\}]$
$+n_2(s)[-m_2(s)T\{u(x, o)\} + n_2(s)T_2\{u(x, y)\}]$
$= n_1(s)n_1(s)T_2\{u(x, y)\} - n_2(s)m_2(s)T\{\sin x\}$
$+ n_2(s)n_2(s)T_2\{u(x, y)\} = 0$

$T_2\{u(x, y)\}[n_1(s)^2 + n_2(s)^2] = n_2(s)m_2(s)[\frac{m1(s)}{n1(s)^2+1}]$

$T_2\{u(x, y)\}[n_1(s)^2 + n_2(s)^2] = [\frac{n2(s)m2(s)m1(s)}{[n1(s)^2 + n2(s)^2][n1(s)^2+1]}$

If $n_2(s)=1$ and $m_1(s)=m_2(s)$, then

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DIALOGUE SOCIAL SCIENCE REVIEW

Vol. 3 No. 12 (December) (2025)

$T_2\{u(x, y)\}[n_1(s)^2 + n_2(s)^2] = [\frac{m2(s)m1(s)}{[n1(s)^2 + 1][n1(s)^2+1]}$

Apply inverse ,we get $\{u(x, y)\} = \sin^2 x$

4.2 Example . $\{\frac{\partial^2 u}{\partial x^2}\} + \{\frac{\partial^2 f}{\partial y^2}\} = 0$

under some restrictions. $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b)$ is equal to zero. and $u(x, 0) = \sin x$

Solution

$T_2\{\frac{\partial}{\partial x} u_x(x, y)\} + T_2\{\frac{\partial}{\partial y} u_y(x, y)\}$

$\Rightarrow -m_1(s)T\{u_x(0, y)\} + n_1(s)T_2\{u_x(x, y)\}$

$-m_2(s)T\{u_x(x, 0)\} + n_2(s)T_2\{u_x(x, y)\}$

$\Rightarrow -m_1(s)T\{u_x(0, y)\} + n_1(s)[-m_1(s)T\{u(0, y)\}$

$+ n_1(s)T_2\{u(x, y)\}] - m_2(s)T\{u_x(x, 0)\}$

$+ n_2[-m_2(s)T\{u(x, 0)\} + n_2(s)T_2\{u(x, y)\}] = 0$

$\Rightarrow -n_1(s)m_1(s)T_2\{u(x, y)\} - m_2(s)[T\{\cos x\}]$

$-n_2(s)m_2(s)T\{\sin x\} + n_1(s)^2 T_2\{u(x, y)\} = 0$

$T_2\{u(x, y)\}[-n_1(s)^2 + n_2(s)^2] = m_2(s)T\{\cos x\} + n_2(s)m_2(s)T\{\sin x\}$

$= m_2(s)[\frac{m1(s)n1(s)}{n1(s)^2+1}] + n_2(s)m_2(s)[\frac{m1(s)}{n1(s)^2+1}]$

$T_2\{u(x, y)\} = \frac{m1(s)m2(s)n1(s)}{[n1(s)^2+1]} + \frac{m1(s)m2(s)n2(s)}{[n1(s)^2+1]}$

$T_2\{u(x, y)\} = \frac{-m1(s)m2(s)[n1(s)+n2(s)]}{[n1(s)^2+1][n1(s)^2-n1(s)^2]}$

$T_2\{u(x, y)\} = \frac{-m1(s)m2(s)}{[n1(s)^2][n1(s)-n2(s)]}$

If $m_1(s) = m_2(s)$

If $n_1(s) = 1$

$\Rightarrow T_2\{u(x, y)\} = -[\sin x][\frac{n_1(s)}{m_1(s)-1}]$

$\Rightarrow \{u(x, y)\} = -e^t \sin x$

5. Conclusion

DGIT aims to streamline the problem into a set of ordinary differential equations, offering greater ease of manipulation. As a valuable asset, DGIT facilitates the resolution of hyperbolic boundary value problems across diverse scientific and engineering domains. By effecting a domain transformation and simplifying the equations, DGIT offers insights into solution behavior

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