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Hermite-Hadamard Like Inequalities for $M-$Convex Function and Implementations

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ABSTRACT
In the present article, we prove the Hermite-Hadamard (H-H) like inequalities for M−convex (conv.) and implementations for theory of probability (prob.) & numerical integrati0n are deduced. Some consequences of several published article would be captured as especial cases. Moreover, we deduce few especial cases of M−conv. function (func.).

Keywords: Convex Func., h-h Inequality (inequal.), Power-Mean Inequal., HÖLder Inequal., Numerical Integration, Prob. Density Func.
2010 MSC. 26A51, 33B15, 26D10, 26D15, 26D20

INTRODUCTION
We must generalize the idea of convex functions in order to generalize Ostrowski's inequality. In this way, we may quickly identify the generalizations and specific

instances of the inequal. We recollect several definitions from the literature [2,3,4,5,8,9,10,11,12,13,14,15,16] for variety of convex functi**0**ns.

Definition 1.1. A func. $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is known as conv., if

$$g(\vartheta v + (1-\vartheta) w) \leq \vartheta g(v) + (1-\vartheta) g(w), \qquad (1.1)$$

$\forall\, w, v \in K$, $\vartheta \in [0,1]$.

We remind term of M−conv. func. (see [6]).

Definition 1.2. Let $M \in [0,1]$. A func. $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is known as M−conv., if

$$g(\vartheta v + M(1-\vartheta) w) \leq \vartheta g(v) + M(1-\vartheta) g(w), \qquad (1.2)$$

$\forall\, w, v \in K$, $\vartheta \in [0,1]$.

Remark 1.4. The terms of standard conv. func. & star-shaped func. are obtained if choose $M = 1$, $M = 0$ in the above inequality respectively.

In practically all scientific fields, inequalities play a major impact. Our primary target is on H–H like inequalities, despite the discipline's enormous scope.

The convexity theory is closely connected to the inequalities theory. Many well-known Inequalities in literature consequence directly from applying conv. functions. The H–H equation is a notable equation for conv. funct. that has been thoroughly researched in recent decades. It is discovered independently by Hadamard & Hermite and it is stated as follows: Any func. $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ be conv., $m, \ell \in K$ with $m > \ell$, If & only if

$$g\left(\frac{\ell+m}{2}\right) \leq \frac{1}{m-\ell}\int_{\ell}^{m} g(v)\,dv \leq \frac{g(\ell)+g(m)}{2} \qquad (1.3)$$

this is kn**0**wn as H-H inequal. Equation (1.3) has become a crucial pillar in the area of prob. & optimization. Additionally, numerous researchers have refined or generalized equation (1.3) for conv., s- conv., quasi- conv., & various other varieties of functi**0**ns.

In [1], the following consequence had derived by Agarwal & Dragomir, which includes the H-H like integral inequal.

Proposition 1.5. Let $g : K^0 \subseteq (-\infty,\infty) \to (-\infty,\infty)$ be differentiable mapping (diff. map.) in the Interior K^o 0f K, here $m, \ell \in K^o$ along $m > \ell$. If $|g'|$ is conv. in interval $[\ell, m]$. Then the below equation holds

$$\left|\frac{g(\ell)+g(m)}{2} - \frac{1}{m-\ell}\int_{\ell}^{m} g(u)\,du\right| \leq \frac{(m-\ell)(|g'(\ell)|+|g'(m)|)}{8} \qquad (1.4)$$

For additional recent c**0**nsequences on H-H like inequalities involving various classes of conv. functions, refer to [7,17,18].

Kavurmaci et al. [7] established few novel inequalities **0**f H-H like f**0**r conv. functions & implementations by utilizing Hölder inequal. and Power-mean inequal.

The primary goal of the article is to generalize few H-H like inequalities to M−conv. func. by employing the Hölder & Powermean inequalities. The implementations also encompass areas such as prob. & numerical integration. We will capture some findings of various articles [1,7] and also examine especial cases of M−conv. func.

HERMITE-HADAMARD LIKE INEQUALITIES FOR M−CONVEX FUNCTION

Regarding proof of our primary findings, below Lemma (see [7]) is required.

Lemma 2.1. Supp0se $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is diff. map. in interval $K^0 \subset (-\infty,\infty)$ here $Mm, M\ell \in K$ along $Mm > M\ell$, if $g' \in L[M\ell, Mm]$, then

$$\frac{(Mm - v)g(Mm) + (v - M\ell)g(M\ell)}{m-\ell} - \frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)\,du$$

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$= \frac{(v-M\ell)^2}{m-\ell}\int_0^1 (1-\vartheta)g'(\vartheta v + M(1-\vartheta)\ell)d\vartheta + \frac{(Mm-v)^2}{m-\ell}\int_0^1 (1-\vartheta)g'(\vartheta v + M(1-\vartheta)m)d\vartheta.$

Proof. We obtain the desired consequence by employing likewise techniques of proof of Lemma 1 of [7].

Remark 2.1. We recapture Lemma 1 of [7] if choose M = 1.

The below consequences may be derived by employing lemma 2.1

Theorem 2.2. Supp0se $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is diff. map. in interval $K^0 \subset (-\infty,\infty)$ ε $g' \epsilon L[M\ell, Mm]$, here $Mm, M\ell \in K$ along $Mm > M\ell$, if $|g'|$ is M−conv. on $[M\ell, Mm]$, then

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell} - \frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\left[\frac{|g'(v)|+2M|g'(\ell)|}{6}\right] + \frac{(Mm-v)^2}{m-\ell}\left[\frac{|g'(v)|+2M|g'(m)|}{6}\right].$$

For every $g\epsilon[M\ell, Mm]$

Proof. Utilizing lemma 2.1 & taking Modulus, then

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell} - \frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\int_0^1 (1-\vartheta)|g'(M(1-\vartheta)\ell + \vartheta v)|d\vartheta + \frac{(Mm-v)^2}{m-\ell}\int_0^1 (1-\vartheta)|g'(\vartheta v + M(1-\vartheta)m)|d\vartheta$$

∵ |g’ | is M−conv., then

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell} - \frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\int_0^1 (1-\vartheta)[\vartheta|g'(v)| + M(1-\vartheta)|g'(\ell)|]\,d\vartheta$$
$$+\frac{(Mm-v)^2}{m-\ell}\int_0^1 (1-\vartheta)\,[\vartheta|g'(v)| + M(1-\vartheta)|g'(m)|]d\vartheta$$
$$= \frac{(v-M\ell)^2}{m-\ell}\left[\frac{|g'(v)|+2M|g'(\ell)|}{6}\right] + \frac{(Mm-v)^2}{m-\ell}\left[\frac{|g'(v)|+2M|g'(m)|}{6}\right]$$

thus completing the proof.

Remark 2.3. The above result is also obtained for term of star-shaped func. if choose M=0 in The**0**rem 2.2.

Remark 2.4. We attain the Theorem 4 of [7] If choose M = 1 in The**0**rem 2.2.

Corollary 2.5. In The0rem 2.2, choosing $v = \frac{M\ell+Mm}{2}$ we get

$$\left|M\frac{g(Mm)+g(M\ell)}{2} - \frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq M^2\frac{(m-\ell)}{12}\left[\left|g'\left(\frac{M\ell+Mm}{2}\right)\right| + M(|g'(\ell)| + |g'(m)|)\right]$$

Remark 2.6. Few Remarks regarding Cor**0**llary 2.5 are below as especial cases.

(i) By utilizing the convexity property of $|g'|$ in Corollary 2.5, we get established equation (1.4) (capture theorem 2.2 of article [1]).

(ii) The above result is also obtained for term of star-shaped func. if choose M=0 in Corollary 2.5.

Remark 2.7. We capture the Corollary 2 of article [7] If choose M = 1 in Corollary 2.5.

Theorem 2.8. Supp0se $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is diff. map. in $K^0 \subset$

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$(-\infty,\infty)$ ε $g' \epsilon L[M\ell, M m]$, here $M\ell, M m \in K$ along $M m > M\ell$. If $|g'|^{\frac{p}{p-1}}$ is M−conv. in the interval $[M\ell, M m]$ & f0r s0me fixed $1 < q$. Then

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell}-\frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[\frac{(v-M\ell)^2}{m-\ell}\left(|g'(v)|^q+M|g'(\ell)|^q\right)^{\frac{1}{q}} + \frac{(Mm-v)^2}{m-\ell}\left(|g'(v)|^q+M|g'(m)|^q\right)^{\frac{1}{q}}\right]$$

for every $g \in [M\ell, M m]$ and $q=\frac{p}{p-1}$.

Proof. Utilizing Lemma 2.1 & M−convexity of $|g'|$ & then implementing the widely recognized Hölder inequality, we get

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell}-\frac{1}{m-\ell}\int_{M\ell}^{Mm} g(u)du\right|$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\int_0^1 (1-\vartheta)|g'(\vartheta v+M(1-\vartheta)\ell)|d\vartheta + \frac{(Mm-v)^2}{m-\ell}\int_0^1 (1-\vartheta)|g'(\vartheta v+M(1-\vartheta)m)|d\vartheta$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\int_0^1 (1-\vartheta)[\vartheta|g'(v)|+M(1-\vartheta)|g'(\ell)|]\,d\vartheta$$
$$+\frac{(Mm-v)^2}{m-\ell}\int_0^1 (1-\vartheta)\,[\vartheta|g'(v)|+M(1-\vartheta)|g'(m)|]d\vartheta$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\left(\int_0^1 (1-\vartheta)^p d\vartheta\right)^{\frac{1}{p}}\left[\int_0^1 (\vartheta|g'(v)|+M(1-\vartheta)|g'(\ell)|)^q d\vartheta\right]^{\frac{1}{p}}$$
$$+\frac{(Mm-v)^2}{m-\ell}\left(\int_0^1 (1-\vartheta)^p d\vartheta\right)^{\frac{1}{p}}\left[\int_0^1 (\vartheta|g'(v)|+M(1-\vartheta)|g'(m)|)^q d\vartheta\right]^{\frac{1}{p}}$$
$$\leq \frac{(v-M\ell)^2}{m-\ell}\left(\int_0^1 (1-\vartheta)^p d\vartheta\right)^{\frac{1}{p}}\left[\int_0^1 \vartheta|g'(v)|^q d\vartheta+M\int_0^1 (1-\vartheta)|g'(\ell)|^q d\vartheta\right]^{\frac{1}{p}}$$
$$+\frac{(Mm-v)^2}{m-\ell}\left(\int_0^1 (1-\vartheta)^p d\vartheta\right)^{\frac{1}{p}}\left[\int_0^1 \vartheta|g'(v)|^q d\vartheta+M\int_0^1 (1-\vartheta)|g'(m)|^q d\vartheta\right]^{\frac{1}{p}}$$
$$\leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[\frac{(v-M\ell)^2}{m-\ell}\left(|g'(v)|^q+M|g'(\ell)|^q\right)^{\frac{1}{q}} + \frac{(Mm-v)^2}{m-\ell}\left(|g'(v)|^q+M|g'(m)|^q\right)^{\frac{1}{q}}\right]$$

Remark 2.9. The above result is also obtained for term of star-shaped func. if choose M=0 in The**0**rem 2.8.

Remark 2.10. We attain the The**0**rem 5 0f [7] If choose M = 1 in Theorem 2.8.

Corollary 2.11. In The0rem 2.8, choosing $v = \frac{M\ell+Mm}{2}$ we get

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$$\left|M\frac{g(Mm)+g(M\ell)}{2}-\frac{1}{m-\ell}\int_{M\ell}^{Mm}g(u)du\right|\le M^2\frac{(m-\ell)}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[\left(M|g'(\ell)|^q+\left|g'\left(\frac{M\ell+Mm}{2}\right)\right|^q\right)^{\frac{1}{q}}+\left(M|g'(m)|^q+\left|g'\left(\frac{M\ell+Mm}{2}\right)\right|^q\right)^{\frac{1}{q}}\right]$$

Remark 2.12. The above result is also obtained for term of star-shaped func. if choose M=0 in Corollary 2.11.

Remark 2.13. We capture the Corollary 3 of article [7] If choose M= 1 in Corollary 2.11.

Theorem 2.14. Supp0se $g : K \subseteq (-\infty,\infty) \to (-\infty,\infty)$ is diff. map. on $K^0 \subset (-\infty,\infty)$ ε $g' \in L[M\ell, Mm]$, here $M\ell, Mm \in K$ along $M\ell < Mm$. If $|g'|^q$ is M−conv. on $[M\ell, Mm]$ & f0r s0me fixed $q \ge 1$. Then

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell}-\frac{1}{m-\ell}\int_{M\ell}^{Mm}g(u)du\right|$$
$$\le\frac{1}{2\,(3)^{\frac{1}{q}}\,(m-\ell)}\Big[(v-M\ell)^2(|g'(v)|^q+2M|g'(\ell)|^q)^{\frac{1}{q}}$$
$$+(Mm-v)^2(|g'(v)|^q+2M|g'(m)|^q)^{\frac{1}{q}}\Big]$$

for every $g \in [M\ell, Mm]$, $q=\frac{p}{p-1}$

Proof. Consider $1 \le q$ & Utilizing Lemma 2.1 & implementing the widely recognized p**0**wer-mean inequality, we get

$$\left|\frac{(Mm-v)g(Mm)+(v-M\ell)g(M\ell)}{m-\ell}-\frac{1}{m-\ell}\int_{M\ell}^{Mm}g(u)du\right|$$
$$\le\frac{(v-M\ell)^2}{m-\ell}\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)\ell)|d\vartheta$$
$$+\frac{(Mm-v)^2}{m-\ell}\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)m)|d\vartheta$$
$$\le\frac{(v-M\ell)^2}{m-\ell}\left(\int_0^1(1-\vartheta)d\vartheta\right)^{1-\frac{1}{q}}\left(\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)\ell|^q d\vartheta\right)^{\frac{1}{q}}$$
$$+\frac{(Mm-v)^2}{m-\ell}\left(\int_0^1(1-\vartheta)d\vartheta\right)^{1-\frac{1}{q}}\left(\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)m|^q d\vartheta\right)^{\frac{1}{q}}$$

∵ |g' | is M−conv. & first we taking term

$$\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)\ell|^q d\vartheta$$
$$\le\int_0^1(1-\vartheta)[\vartheta|g'(v)|^q+M(1-\vartheta)|g'(\ell)|^q]\,d\vartheta$$
$$=\frac{|g'(v)|^q+2M|g'(\ell)|^q}{6}$$

Analogously,

$$\int_0^1(1-\vartheta)|g'(\vartheta v+M(1-\vartheta)m|^q d\vartheta\le\frac{\left|g'^{(v)}\right|^q+2M\left|g'^{(m)}\right|^q}{6}.$$

We obtain the desired result by combining all above inequalities.

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Remark 2.15. The above result is also obtained for term of star-shaped func. if choose M=0 in The0rem 2.14.

Remark 2.16. We attain the Theorem 7 0f article [7] If choose M = 1 in The0rem 2.14.

Corollary 2.17. In The0rem 2.14, choosing $\mathcal{v} = \frac{M\ell+M\mathcal{m}}{2}$ we get

$$\left|M\frac{g(M\mathcal{m})+g(M\ell)}{2} - \frac{1}{\mathcal{m}-\ell}\int_{M\ell}^{M\mathcal{m}} g(u)du\right|$$
$$\leq M\frac{(\mathcal{m}-\ell)}{8\,(3)^{\frac{1}{q}}}\left[\left(\left|g'\left(\frac{M\ell+M\mathcal{m}}{2}\right)\right|^q + 2M|g'(\ell)|^q\right)^{\frac{1}{q}} + \left(\left|g'\left(\frac{M\ell+M\mathcal{m}}{2}\right)\right|^q + 2M|g'(\mathcal{m})|^q\right)^{\frac{1}{q}}\right]$$

Remark 2.18. The above result is also obtained for term of star-shaped func. if choose M=0 in Corollary 2.17.

Remark 2.19. We capture the Corollary 4 of article [7] If choose M= 1 in Corollary 2.17.

IMPLEMENTATION TO NUMERICAL INTEGRATION

The Trapez0idal F0rmula. Supp0se d : $M\ell = M\theta_0 < M\,\theta_1 < \cdots < M\theta_n = M\mathcal{m}$ is divisi0n 0f interval $[M\ell, M\mathcal{m}]$ & $h_i = \theta_{i+1} - \theta_i$, $(i = o,1,2,\cdots, n-1)$ & c0nsider the quadrature f0rmula

$$\int_{M\ell}^{M\mathcal{m}} g(u)du = Q(g,d) + R(g,d), \qquad (3.1)$$

where

$$Q(g,d) = \sum_{i=0}^{n-1}\left((M\theta_{i+1}-\mathcal{v})g(M\theta_{i+1}) + (\mathcal{v}-M\theta_i)g(M\theta_i)\right)$$

for the trapez0idal versi0n & R(g,d) represents the ass0ciated approximati0n err0r.

Theorem 3.1. With all the suppositions of The0rem 2.2, f0r every divisi0n d of the interval $[M\ell, M\mathcal{m}]$. Then in equation (3.1), the trapez0idal err0r estimate satisfies:

$$|R(g,d)|$$
$$\leq \sum_{i=0}^{n-1}(\mathcal{v}-M\theta_i)^2\left[\frac{|g\prime(\mathcal{v})|+2M|g\prime(\theta_i)|}{6}\right] + \sum_{i=0}^{n-1}(M\theta_{i+1}-\mathcal{v})^2\left[\frac{|g\prime(\mathcal{v})|+2M|g\prime(\theta_{i+1})|}{6}\right]$$

for every $g \in [M\ell, M\mathcal{m}]$.

Proof. Utilizing the0rem 2.2 on sub-interval $[\theta_i, \theta_{i+1}]$ here $i = o,1,2, \ldots, n-1$ of the divisi0n, we get

$$\left|\frac{(M\theta_{i+1}-\mathcal{v})g(M\theta_{i+1}) + (\mathcal{v}-M\theta_i)g(M\theta_i)}{h_i} - \frac{1}{h_i}\int_{M\theta_i}^{M\theta_{i+1}} g(u)du\right|$$
$$\leq \frac{(\mathcal{v}-M\theta_i)^2}{h_i}\left[\frac{|g\prime(\mathcal{v})|+2M|g\prime(\theta_i)|}{6}\right] + \frac{(M\theta_{i+1}-\mathcal{v})^2}{h_i}\left[\frac{|g\prime(\mathcal{v})|+2M|g\prime(\theta_{i+1})|}{6}\right]$$

Now we take summation over i in equation (3.1) from 0 to n − 1, then

$$\left|\int_{M\ell}^{M\mathcal{m}} g(u)du - Q(g,d)\right| = \left|\sum_{i=0}^{n-1}\left[\int_{M\theta_i}^{M\theta_{i+1}} g(u)du - \left((M\theta_{i+1}-\mathcal{v})g(M\theta_{i+1}) + (\mathcal{v}-M\theta_i)g(M\theta_i)\right)\right]\right|$$
$$\leq \sum_{i=0}^{n-1}\left|\int_{M\theta_i}^{M\theta_{i+1}} g(u)du - \left((M\theta_{i+1}-\mathcal{v})g(M\theta_{i+1}) + (\mathcal{v}-M\theta_i)g(M\theta_i)\right)\right|$$
$$\leq \sum_{i=0}^{n-1}(\mathcal{v}-M\theta_i)^2\left[\frac{|g'(\mathcal{v})|+2M|g'(\theta_i)|}{6}\right] + \sum_{i=0}^{n-1}(M\theta_{i+1}-\mathcal{v})^2\left[\frac{|g'(\mathcal{v})|+2M|g'(\theta_{i+1})|}{6}\right]$$

which completes the proof.

Remark 3.2. We can also derive above similar findings for Theorem 2.14, Corollary 2.17, Cor0llary 2.11, The0rem 2.8 & Cor0llary 2.5.
Remark 3.3. Entire Remarks of 2nd secti0n are als0 h0ld for Remark 3.2 & Theorem 3.1.

IMPLEMENTATION TO PROBABILITY THEORY

Supp0se $\mathfrak{R}$ is a rand0m variable taking values in the finite $[M\ell, Mm]$ with the prob. density func. $g : [M\ell, Mm] \to [0,1]$ & the cumulative distribution func. $\mathcal{G}(v) = P(\mathfrak{R} \le v) = \int_{M\ell}^{Mm} g(u)du$.

Theorem 4.1. With all the assumpti0ns 0f The0rem 2.2, then

$$\left|\frac{(Mm - v)\mathcal{G}(Mm) + (v - M\ell)\mathcal{G}(M\ell)}{m - \ell} - \frac{Mm - \mathcal{E}(\mathfrak{R})}{m - \ell}\right| \le \frac{(v-M\ell)^2}{m-\ell}\left[\frac{|\mathcal{G}'(v)|+2M|\mathcal{G}'(\ell)|}{6}\right] + \frac{(Mm-v)^2}{m-\ell}\left[\frac{|\mathcal{G}'(v)|+2M|\mathcal{G}'(m)|}{6}\right] \quad (4.1)$$

for every $g \in [M\ell, Mm]$. Here $\mathcal{E}(\mathfrak{R})$ is the expectati0n of $\mathfrak{R}$.
Pr00f. Choose $g = \mathcal{G}$, we attain (4.1), by implementing the below identity in Theorem 2.2.

$$\mathcal{E}(\mathfrak{R}) = \int_{M\ell}^{Mm} u\mathcal{G}(u)du = Mm - \int_{M\ell}^{Mm} \mathcal{G}(u)du.$$

$\because \mathcal{G}(M\ell) = 0$ & $\mathcal{G}(Mm) = 1$.

Remark 4.3. Entire Remarks 0f 2nd secti0n are als0 h0ld f0r Remark 4.2 & Theorem 4.1.

CONCLUSION

In the current article, the famous H-H like inequalities f0r M−conv. func. are obtained by implementing the widely recognized power-mean & Hölder's inequalities & implementations for theory of prob. & numerical integrati0n are also deduced. We have captured various findings of several published articles [1,7] & also deduced few especial cases of M−conv. func.

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